ELSEVIER

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Earth and Planetary Science 5 (2012) 213 – 217

Procedia
Earth and Planetary Science

2012 International Conference on Structural Computation and Geotechnical Mechanics

Application of Improved Metabolic GM Model in Analyzing Settlement of Earth-rock Dam in the Period of Construction

NIU Jingtai[a,b], ZHOU Bin[c], PENG Youwen[a], WEI Bowen[b], a*

[a]*Nanchang institute of technology, Nanchang 330099,China*
[b]*College of Water-conservancy and Hydropower, Hohai University, Nanjing 210098, China*
[c]*Lincang Survey and Design Institute of Water Resource and Hydraulic, Lincang 677000, China*

Abstract

When analyzing the settlement which is influenced by multi-factors such as speed up construction, sharp increase in load and rainfall etc. in the period of earth-rock dam, the disturbance factors that continue to enter the system should be considered, and the metabolic GM (1, 1) model is introduced to improve the fitting and prediction accuracy. Because the background value of the traditional GM (1, 1) model has some deviation, this paper further improves the fitting and prediction accuracy through making the calculated method of background value better. Finally, the effectiveness of improved metabolic GM (1, 1) model is verified through an example.

Keywords: earth-rock dam; construction period; analysis of settlement; improved metabolic GM model

1. Introduction

Subsidence monitoring to earth-rock dam during the period of construction is of great importance [1]. Many scholars at home and abroad believe that the Grey Prediction can be applied to the settlement analysis of earth-rock dam because of the influence of multiple factors. By using the GM(1,1) model prediction[2,3], only a handful of data with higher accuracy behind the origin can be obtained, while with the further process, the accuracy of data will get worse, even large deviation emerges and eventually fails completely. The reason exists in some random disturbances and driving factors, which especially produced by the speed up construction and sharp increase in load and rainfall etc, continuing effect the

* Corresponding author: NIU Jingtai. Tel.: +86-791-82086424.
E-mail address: niujingtai@163.com.

doi:10.1016/j.proeps.2012.01.037

system. Compared to the GM (1, 1) model, the metabolic GM (1,1) model[4,5], which removes the old information and add new information has more accuracy in prediction. The latter model makes the new data sequence better reflect the variation law of the system. In the metabolic GM (1, 1) model, the prediction accuracy is directly affected by the development coefficient and grey action which determine the background value thus the higher accuracy of the background implies the higher accuracy of the model [6].

2. Establishment of improved metabolic GM (1,1) model

2.1. Establishment of GM (1,1) model

We define settlement sequence $X^{(0)}$ under the condition of equal interval as follows:

$$X^{(0)} = \left\{x^{(0)}_{(1)}, x^{(0)}_{(2)}, x^{(0)}_{(3)}, \cdots x^{(0)}_{(n)}\right\} \tag{1}$$

By 1-AGO transfer according to the grey system theory, then the new sequence is written as:

$$X^{(1)} = \left\{x^{(1)}_{(1)}, x^{(1)}_{(2)}, x^{(1)}_{(3)}, \cdots x^{(1)}_{(n)}\right\}, \quad X^{(1)}_{(i)} = \sum_{i=1}^{k} x^{(0)}_{(k)}, x^{(0)}_{(1)} = x^{(1)}_{(1)} \quad (k = 1\,2\,3 \cdots n) \tag{2}$$

Let $Z^{(1)}$ be background value of $X^{(1)}$: $Z^{(1)}_{(k)} = \left\{Z^{(1)}_{(2)}, Z^{(1)}_{(3)}, Z^{(1)}_{(4)} \cdots \cdots Z^{(1)}_{(n)}\right\}$. The grey differential equations of GM (1, 1) model is given by: $x^{(0)}_{(k)} + aZ^{(1)}_{(k)} = b$. Given two matrices B and Y:

$$B = \begin{bmatrix} -Z^{(1)}_{(2)} & -Z^{(1)}_{(2)} & \cdots & -Z^{(1)}_{(n)} \\ 1 & 1 & \cdots & 1 \end{bmatrix}^T, \quad Y = \begin{bmatrix} x^{(0)}_{(2)} & x^{(0)}_{(n)} & \cdots & x^{(0)}_{(n)} \end{bmatrix}^T \tag{3}$$

The coefficient vector $\hat{a} = (a,b)^T$ is founded by least square method as follows: $\hat{a} = (B^T B)^{-1} B^T Y$.

Thus the corresponding grey differential equations of GM (1,1) model is given by: $dx^{(1)}_{(t)} / dt + ax^{(1)}_{(t)} = b$. And then let us write the whiten equations above as $x^{(1)}_{(t)} = (x^{(1)}_{(1)} - b/a)e^{-at} + b/a$ .Discretizing the whiten equations, we obtain prediction model of GM (1,1) model:

$$\hat{x}^{(1)}_{(k+1)} = \left(x^{(0)}_{(1)} - b/a\right)e^{-ak} + b/a \quad (k = 1\,2\,3 \cdots n) \tag{4}$$

After reducing treat (4), we outputs the predictive value of original sequence:

$$\hat{x}^{(0)}_{(k+1)} = \hat{x}^{(1)}_{(k+1)} - x^{(1)}_{(k)} = \left(1 - e^{a}\right)\left[x^{(0)}_{(1)} - b/a\right]e^{-ak} \quad (k = 1\,2\,3 \cdots n) \tag{5}$$

2.2. Improved background value $Z^{(1)}_{(k)}$

We used to calculate background value by the traditional method, which is written as $Z^{(1)}_{(k)} = 0.5\left(x^{(1)}_{(k)} + x^{(1)}_{(k-1)}\right)$, but according to the actual meaning of background value, considerable errors would be produced. In fact, the exact background value is $\int_{k-1}^{k} x^{(1)}_{(t)} dt \quad (k = 2\,3\,4 \cdots n)$. In order to gain the accurate value, we define $x^{(1)}_{(k)} = Ae^{\beta k} + B$, where $A = -Ce^{\beta} / \left(1 - e^{\beta}\right)$, $B = -A = Ce^{\beta} / \left(1 - e^{\beta}\right)$, that is,

$x_{(k)}^{(0)} = x_{(k)}^{(1)} - x_{k-1}^{(1)} = Ce^{\beta k}$.

Inserting $x_{(k)}^{(1)} = Ae^{\beta k} + B$ into formula of the exact background value:

$$Z_{(k)}^{(1)} = \int_{k-1}^{k}\left(Ae^{\beta t} + B\right)dt = \left(Ae^{\beta k} - Ae^{\beta(k-1)}\right)/\beta + B = \left(x_{(k)}^{(1)} - x_{(k-1)}^{(1)}\right)/\beta + B = x_{(k)}^{(0)}/\beta + B \tag{6}$$

We note $x_{(k)}^{(1)}/x_{k-1}^{(0)} = e^{\beta}$ and recall $x_{(k)}^{(0)} = Ce^{\beta k}$. Thus, some parameters yield as fellow:

$$\beta = \ln x_{(k)}^{(0)} - \ln x_{(k-1)}^{(0)}\ , \quad C = x_{(k)}^{(0)}\left[x_{(k)}^{(0)}/x_{(k-1)}^{(0)}\right]^{-k} \tag{7}$$

$$B = -A = C\left(e^{-\beta} - 1\right)^{-1} = C\left[1/e^{\beta} - 1\right]^{-1} = \left[x_{(k-1)}^{(0)}\right]^{k}/\left[x_{(k)}^{(0)}\right]^{(k-2)}\left[x_{(k-1)}^{(0)} - x_{(k)}^{(0)}\right] \tag{8}$$

Inserting formula (7) and formula (9) into formula (6). Let us write the solution $Z_{(k)}^{(1)}$ as:

$$Z_{(k)}^{(1)} = x_{(k)}^{(0)}/\ln x_{(k)}^{(0)} - \ln x_{(k-1)}^{(0)} + \left[x_{(k-1)}^{(0)}\right]^{k}/\left[x_{(k)}^{(0)}\right]^{(k-2)}\left[x_{(k-1)}^{(0)} - x_{(k)}^{(0)}\right] \tag{9}$$

2.3. Metabolic process of improved GM (1,1) model

During the process of subsidence data acquisition of the earth-rock in its construction, information significance of the earlier data will be weaken gradually because of the effects of some adverse factors, such as high speed construction, shapely load increase, rainfall, etc. It is necessary to insert the new coming data which contain the random disturbance into the data sequence for the prediction so that the changing trend of settlement can be better reflected. It is obviously reasonable to replace each useless earlier datum by a new datum so that the total number of the data sequence will be unchanged. Simply speaking, the so called metabolic process is a procedure of data updating, its corresponding model is named as metabolic model.

Let us define the original settlement data sequence as $X_1^{(0)} = \left\{x_{(1)}^{(0)}, x_{(2)}^{(0)}, x_{(3)}^{(0)} \cdots x_{(n)}^{(0)}\right\}$, after removing the old data and adding new data, we obtain the new data sequence $X_2^{(0)} = \left\{x_{(2)}^{(0)}, x_{(3)}^{(0)}, x_{(4)}^{(0)} \cdots x_{(n+1)}^{(0)}\right\}$.

Based on model GM (1, 1) the prediction model of these two sequences above-mentioned can be written respectively as follows:

$$\hat{x}_{(k+1)}^{(1)} = \left(x_{(1)}^{(0)} - b_1/a_1\right)e^{-ak} + b_1/a_1\ ,\ \hat{x}_{2(k+1)}^{(1)} = (x_{(2)}^{(0)} - b_2/a_2)e^{-ak} + b_2/a_2 \quad \left(k = 1\,2\,3 \cdots n\right) \tag{10}$$

The modeling idea of metabolic GM (1, 1) is embodied through the modeling prediction process above. This process indicates that the parameter values after a certain time can be calculated by the previous data sequence. That means the model has good predictability. The control model also has strong adaptability because for each new data (information), there is a related new model. Since $\left(a_1, b_1\right)$ is not completely equal to $\left(a_2, b_2\right)$, the control system is time-varying self-adaptive. The process of updating information ensures the effectiveness and real-time performance, and then the changes of system behavior can be better reflected.

2.4. Accuracy inspection for improved metabolic GM(1,1) model

In order to inspect the accuracy of improved metabolic GM (1, 1) model, we need test the posterior error of the model, in other words, calculate the deviation S_1 of observed data and the deviation S_2 of residual as follows:

$$S_1^2=\sum_{k=1}^{n}\left(x_{(k)}^{(0)}-\bar{x}\right)^2 \Big/ n\ ,\ S_2^2=\sum_{k=1}^{n}\left(\varepsilon_{(k)}^{(0)}-\bar{\varepsilon}\right)^2 \Big/ n,\ \bar{x}=\sum_{k=1}^{n}x_{(k)}^{(0)} \Big/ n\ ,\ \bar{\varepsilon}=\sum_{k=1}^{n}\varepsilon_{(k)}^{(0)} \Big/ n\ ,\ \varepsilon_{(k)}^{(0)}=x_{(k)}^{(0)}-\hat{x}_{(k)}^{(0)} \qquad (11)$$

The ratio of posterior error is $C=S_2/S_1$, while the probability of minimum error is $P=\left\{\left|\varepsilon_{(k)}^{(0)}-\bar{\varepsilon}\right|<0.6745S_1\right\}$. Depended on the ration C of posterior error and probability P, the model can be diagnosed. The accuracy can be classified into four levels with the magnitude of C and P[6].

3. Case Study

Here we present a sample which is related to a earth-rock dam in its construction period to show how we establish the improved metabolic $GM(1,1)$ model and use it to perform analysis. This dam is a rolling earth-rock dam with 50.5m maximum height, constructed in 1999 and finished in 2001. In order to monitor the settlement of dam foundation in its construction period, observation points are set respectively in left and right dam foundation. Here we select the observed data of left dam foundation as the original data to build the traditional $GM(1,1)$ model, the metabolic $GM(1,1)$ and improved metabolic $GM(1,1)$ models respectively. The calculated results are listed in table 2. Table 1 indicates that $GM(1,1)$ model can be applied in residual prediction analysis because all the three models can result in good accuracies for the earth-rock dam in its construction period. We also can infer from table 1 that the metabolic $GM(1,1)$ model has much higher accuracy than the traditional $GM(1,1)$ model because the former takes account of the continuous interferences to the system. Furthermore, the improved metabolic $GM(1,1)$ model has the best accuracy due to its process that not only includes the effects of continuous interferences, but also corrects the errors produced by the background values in the process of calculation. In summary, the improved metabolic $GM(1,1)$ model can significantly increase the accuracy of fitting and prediction in the process of settlement analysis to the earth-rock dam in its construction period.

Table 1 Three $GM(1,1)$ model

Date	Data	GM-Model		Metabolic GM-Model		Improved Metabolic GM-Model	
		Fitted values *Pre-values	Residul	Fitted values *Pre-values	Residul	Fitted values *Pre-values	Residul
1999/3/29	37.18	37.18	0	-	-	-	-
1999/4/27	38.01	38.44	-0.43	38.01	0	38.01	0
1999/5/28	38.73	38.96	-0.23	38.82	-0.09	38.79	-0.06
1999/6/27	39.95	39.69	0.26	39.76	0.19	39.82	0.13
1999/7/29	40.13	40.35	-0.22	40.29	-0.16	40.25	-0.12
1999/8/29	41.02	40.43	0.59	40.65	0.37	40.79	0.23
1999/9/25	41.41	41.6	-0.19	41.49	-0.08	41.46	-0.05
1999/10/27	41.79	*41.54	0.25	41.6	0.19	41.68	0.11
1999/10/26	42.12	*41.94	-0.19	*42.01	0.11	*42.06	0.06
C		0.242		0.215		0.139	
P		1		1		1	

4. Conclusion

The improved metabolic GM (1, 1) model can increase the accuracy of settlement fitting and prediction for the earth-rock dam in its construction period because the model takes account of the continuous interference factors to the system, such as high speed construction, sharp increase load and rainfall etc. The errors produced by background values are corrected by the way presented in this paper, thus the accuracy of metabolic GM (1, 1) has further improved. The application of the improved metabolic GM (1, 1) model to a sample problem shows this model has much higher accuracy than the other two GM(1,1) models. The improved metabolic GM (1, 1) model not only can be used to analyze settlement of earth-rock dam in its construction period but also can be used to analyze settlement of other buildings in their construction period.

Acknowledgements

This research was financially supported by the science and technology project of Jiangxi province (Grant Nos.2010BSA16800).

References

[1] HUANG Ming, LIU Jun. Settlement-fill monitoring Grey model of embankment Dam[J]. Journal of Shanghai Jiaotong University. 2005, 39(5):786-789.
[2] DENG Julong. Basis of gray theory[M]. Huazhong University of Science and Technology Press, 2002.
[3] LIU Sifeng, XIE Naiming. Grey system theory and its application[M]. Science Press, 2008.
[4] BIAN Peisong, WANG Dengjie, YU Shaohua. Application of the metabolic GM(1,1) model in forecast of buildings subsidence[J]. Journal of Shandong University: Engineering Science, 2010, 40(3):119-123.
[5] WU Xiaoguang, LU Wenquan. Control of asphalt pavement quality based on grey system theory[J]. Journal of Chongqing Jiaotong University, 2004, 23 (2):55-58.
[6] QIAO Xiaoqin. Study on comprehensive evaluation model of Earth-rock dam safety monitoring based on grey theory[D]. Zhengzhou University, 2010.